Mathematical methods and fluid mechanics **(MST326)** content listing

Unit 1 *Properties of fluid*	Definition of viscosity, density, and pressure. Perfect gas law, basic equation of fluid statics, surface forces, and Archimedes' Principle. Boyle's law and basic modelling of the atmosphere.
Unit 2 *Ordinary differential equations*	Cauchy-Euler equations, reduction of order and variation of parameters methods, boundary-value problems, method of power series.
Unit 3 *First-order partial differential equations*	Integration of partial derivatives, chain rule for partial derivatives, method of characteristics.
Unit 4 *Vector field theory*	Cylindrical and spherical polar coordinates. Gradient of scalar fields, divergence and curl of vector fields. Surface and volume integrals. Gauss' theorem and the continuity equation. Stokes' theorem and irrotational vector fields.
Unit 5 *Kinematics of fluids*	Definition of pathlines and streamlines. The stream function and its applications. Modelling fluid motion and the total derivative. The continuity equation and incompressible flows. Derivation of Euler's equation.
Unit 6 *Bernoulli's equation*	Inviscid flows, Bernoulli's equation and its applications: flow through and orifice, and a contraction in a pipe. Open channel flows: classification of flows and flow over a weir.
Unit 7 *Vorticity*	Definition of vorticity, circulation and line vortex. Inviscid flow around an obstacle. Kelvin's theorem and its applications. Introduction to viscous flows, the Reynolds number, and turbulence.
Unit 8 *The flow of a viscous fluid*	Newtonian fluids, flow between parallel plates. The Navier-Stokes equations and boundary conditions. Approximations of the Navier-Stokes equations and some applications: pipe flow and the slider bearing.
Unit 9 *Second-order partial differential equations*	Classification of partial differential equations and method of characteristics. The wave equation and its d'Alembert's solution.
Unit 10 *Fourier series*	Separation of variables. Fourier sine/cosine series, and generalised Fourier series. Sturm-Liouville problems, Legendre series.
Unit 11 *Laplace's equation*	Applications of Laplace's equations, boundary conditions. Rectangular domains and the Principle of superposition. Circular regions, Poisson's integral formula and mean-value theorem, application to flow past a cylinder. Laplace equation in spherical polar coordinates.
Unit 12 *Water waves*	Simple model for water waves, progressive wave solution. Deep water gravity waves, waves in finite depth. Dispersive waves.
Unit 13 *Boundary layers and turbulence*	Definition of boundary layer and laminar/turbulence transition. Laminar boundary layers, momentum integral equation. Turbulent flow, Reynolds averaged Navier-Stokes equations, turbulent boundary layers, friction law for smooth pipes, Colebrook-White equation, and the Moody diagram.